Simulations
Is Martin Improving?
Lesson 38-1 Devising Simulations

ACTIVITY 38

My Notes

Learning Target:

- Devise a simulation that can help determine whether observed data are consistent or inconsistent with a conjecture about how the data were generated.

SUGGESTED LEARNING STRATEGIES: Close Reading, Predict and Confirm, Summarizing, Paraphrasing, Think Aloud, Debriefing, Discussion Groups

Martin enjoys playing video games. On his birthday he received "Man vs. Monsters," a game in which the player plays the role of a person who is trying to save the earth from an invasion of alien monsters. At the end of the game, the player either wins or loses. The first three times Martin played the game, he lost. In the next seven games that he played, he won four times, and he felt like his performance was improving. In fact, the sequence of Martin's wins and losses is as follows, where "L" represents losing a game, and "W" represents winning a game.

L, L, L, W, L, L, W, L, W, W

Martin concluded he was getting better at the game the more he played, and he said that this sequence of wins and losses was evidence of his improvement. His sister Hannah, however, was not convinced. She said, "That sequence of wins and losses looks like a random list to me. If you were really getting better, why didn't you lose the first six and then win the last four?"

In this activity, you will use a ***simulation*** to decide who is correct, Martin or Hannah.

Start by considering that Hannah is correct and that Martin was not really getting better. He had six losses and four wins in a particular order and, if Hannah is correct, those wins and losses could have been arranged in any other order. According to Hannah, Martin's results indicate how good he is at the game—he wins about 40% of the time—but do not indicate whether he is improving.

1. Following this page are ten squares, six of which are marked "Lose" and four of which are marked "Win." These represent the outcomes of the ten games Martin played. Cut out the squares and arrange them *facedown* on your desk.

MATH TERMS

A simulation is a process to generate imaginary data, often many times, using a model of a real-world situation.

Common Core State Standards for Activity 38

HSS-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

ACTIVITY 38
Investigative

Activity Standards Focus

Students are introduced to simulations and how they can be used to determine the truth of a conjecture. Students often enjoy performing simulations like the one in this activity, but you should make sure that they understand *why* they're doing the simulation and what it means.

Lesson 38-1

PLAN

Materials:

- scissors (to cut out squares, or you can make a classroom set of squares to be used in this and the next lesson)

Pacing: 1 class period

Chunking the Lesson

Introduction	#1–2	#3–4
#5–6	#7	#8

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to explain in one sentence what it means to simulate. Have them give an example of a situation that represents a simulation.

Introduction Shared Reading, Summarizing, Paraphrasing, KWL Chart Use reading strategies to be sure students understand the scenario that Martin wins 40% of the time. Discuss how to determine if he is improving and methods of deciding improvement. Students will use a simulation to represent the real world scenario and determine consistency between a model and the real world.

1–2 Create Representations Students will need to cut out the Win/Lose cards on the next page. Use the cards to simulate Martin's wins and losses on the video game. Students will list this sequence. Ask students to determine the reasonableness of their answers.

Teacher to Teacher

Students will need to understand that the pattern when Martin is actually playing the game is a real-world situation, but the random win/loss cards drawn are based solely on selecting cards from the pile.

3–4 Quickwrite, Look for a Pattern, Discussion Groups Students will explain the meaning of the simulations they created in the context of Martin improving at his new video game. It is important to have a number to describe this relationship so that improvement can easily be recognized. Students should discuss and analyze their data by comparing it to others in the class as well as comparing the simulations, which are random. These are not necessarily based on improvements since he is not winning or losing.

Teacher to Teacher

Students are prone to confuse simulations with generating *real* data. It's important for them to understand that by mixing up cards and placing them in a random order, they're generating "data" that are categorically different from Martin's data. Martin's data was real, but the student data sets are all imaginary, generated under a "what-if" scenario.

My Notes

2. Once you have placed the cards facedown, mix them up and arrange them in a random sequential order so that you do not know which ones represent wins and which ones represent losses. Then turn them all face up so you can see the L or W, and write down the order of wins and losses here. This is a simulation of Martin's wins and losses.
Sample answer: W, W, L, L, L, W, L, W, L, L

3. Consider the following two sequences, and write a sentence explaining whether it appears that Martin is improving.

a. L, L, L, L, W, L, L, W, W, W
Martin appears to be improving since the wins are grouped towards the later attempts at playing the game.

b. W, L, W, W, L, L, L, W, L, L
Martin does not appear to be improving since wins are grouped toward the earlier attempts and losses appear to be grouped toward the later games.

4. It is desirable to quantify (i.e., measure with a numerical quantity) the extent to which a sequence of wins and losses indicates that a player who achieved it is really improving. Describe a method that may quantify the results of playing ten games such that the number describes the improvement of a player. Be creative!
Answers will vary. Students may choose to group plays into halves (first and second half) or thirds (beginning, middle, last) and compare numbers of wins. Encourage students to devise a method that would use a single number as a descriptor of improvement to segue into the next question.

My Notes

Lose	Lose
Lose	Lose
Lose	Lose
Win	Win
Win	Win

Teacher to Teacher

Have students cut out these cards. Alternatively, you can make a classroom set of these squares to be used in this and the next lesson.

ACTIVITY 38
continued

My Notes

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Lesson 38-1
Devising Simulations

One way to quantify improvement is to count how many wins occur among the last five games, and subtract the number of wins that occur among the first five games. Call this the "improvement score" for the sequence of results for ten games.

5. Two example sequences are provided below. Find the improvement score for each. Show all work.

L, L, L, L, W, L, L, W, W, W

3 − 1 = 2

W, L, L, W, L, L, L, W, L, W

2 − 2 = 0

6. Using this method to quantify improvement, what would a negative improvement score imply? What would a positive improvement score imply?

A negative improvement score would indicate that the game player had more losses in the last five games than in the first five. This implies that the player was getting worse at the game and was not improving. A positive score would indicate that the player had more wins in the last five games than the first five. This implies that the player is improving.

Any number that summarizes data in a meaningful way is called a ***statistic***. Your improvement score, a number which is the difference between the number of wins among the last five of ten games and the number of wins among the first five, is a statistic because it summarizes the data with a number that measures improvement.

7. Compute the improvement score for the sequence of wins and losses from your simulation in Item 2 when you mixed up the order of your ten squares.

Answers will vary as game sequences will differ.
One possible answer:

W, W, L, L, L, W, L, W, L, L → (2 wins) − (2 wins) = 0

My Notes

MATH TERMS

A **statistic** is a number that summarizes data in a meaningful way. The mean of a data set is an example of a statistic.

5–6 Create Representations, Look for a Pattern Students may have difficulty understanding how to determine if Martin is improving as he plays more often. This gives students a manageable method of analyzing the data and making sense of it.

7 Look for a Pattern This methodology allows students to analyze the data that they gathered through their simulation. Students may have difficulty understanding the differences in the sequences among their classmates. Be sure to stress that each person created a unique sequence by selecting cards randomly.

Developing Math Language

Students need to understand that a summary value that gives meaning to data is called a *statistic*.

8 Summarizing, Paraphrasing, Create Representations, Look for a Pattern Students will use the original data and pattern to see if Martin actually improved.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand how to use quantitative data to determine improvement statistics.

Answers

9. a. This is not a measure of improvement.
   b. No; larger values would not indicate that Martin was improving; only that he was not playing very well initially. He might continue playing equally poorly.
10. a. While somewhat difficult to implement, this is a valid measure of improvement, because steeper slopes would tend to occur when Martin's wins were later in the sequence.
    b. It is also likely to provide more information than Martin's initial statistic because it takes into account the order of the wins, not just the grouping into "first five" and "last five".

My Notes

Recall the reason for computing the improvement score. Martin's sister, Hannah, is skeptical that Martin's ability to win the game is improving. She thinks that his particular sequence of wins and losses looks random and does not imply improvement. To address her concern, it is important to determine whether a sequence like Martin's might easily show up if the order of wins and losses really is random. More specifically, it is important to determine if the improvement score that results from Martin's sequence of wins and losses is a number that might easily result from a random arrangement of four wins and six losses.

8. Compute the improvement score for Martin's actual sequence of wins and losses:

L, L, L, W, L, L, W, L, W, W

(3 wins) − (1 win) = 2

Check Your Understanding

In Item 4, you created a statistic to measure improvement. Below are two other possible "improvement statistics" that Martin might have used to measure his improvement over ten games. For each one, state (a) whether the statistic is actually a measurement of improvement and (b) whether the statistic is likely to provide more information than Martin's improvement score as defined before Item 5. Explain your answers briefly.

9. Count the number of games until Martin achieves his second win. This number of games is the improvement statistic.

10. Identify each win with a "1" and each loss with a "0". Create ordered pairs such that the number of the game (1 through 10) is the x-coordinate and the "1" or "0" is the y-coordinate. Make a scatter plot of these ten points and compute the slope of the regression line through the ten points. The slope of the regression line is the improvement statistic.

Lesson 38-1
Devising Simulations

My Notes

LESSON 38-1 PRACTICE

Teresa conducted a survey of a simple random sample of ten customers shopping in a grocery store. Her survey asked the customers to identify the price of the most expensive item in their basket. The ten responses, rounded to the nearest dollar, are listed below.

12, 8, 3, 2, 9, 25, 14, 8, 4, 5

11. Identify two statistics that could be calculated from these data.

12. Calculate the statistics that you identified in Item 11, and describe the significance of each statistic.

Steven would like to create simulations that would model the incidence of precipitation in a particular city.

13. Consider a fictional city where data indicate that precipitation occurs on 50% of the days in a year. Describe how Steven could perform a simulation to determine the occurrences of precipitation in this city during eight randomly chosen days of the year, using a fair coin.

14. Sacramento, California, receives rain on approximately one of every six days during a year. Describe a method by which Steven may simulate precipitation in Sacramento for eight randomly chosen days of the year.

15. Vero Beach, Florida, receives rain on approximately one of every three days during a year. Describe a method by which Steven may simulate precipitation in Vero Beach for eight randomly chosen days of the year.

16. Hilo, Hawaii receives rain on approximately three of every four days during a year. Describe a method by which Steven may simulate precipitation in Hilo for eight randomly chosen days of the year.

15. Answers will vary. One such simulation would involve writing "precipitation" on one of three identical pieces of paper and "no precipitation" on the other two pieces of paper. Select a paper at random from a container with all three pieces of paper eight times, with replacement, and record the number of times "precipitation" appears in those eight trials. Another such simulation might involve a number cube. Let two numbers indicate precipitation and the other four indicate no precipitation. Roll the number cube eight times for the eight days and record the number of times the precipitation numbers occur.

16. Answers will vary. One such simulation would involve writing "precipitation" on three of four identical pieces of paper and "no precipitation" on the other piece of paper. Select a paper at random from a container with all four pieces of paper eight times, with replacement, and record the number of times "precipitation" appears in those eight trials.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 38-1 PRACTICE

11. Possible statistics may include mean, median, standard deviation, IQR, or range.

12. If statistics of mean (9) or median (8) are chosen, then significance involves finding a measure of center. If standard deviation (6.82), IQR (8), or range (23) is chosen, the significance involves a measure of spread.

13. Flip the coin eight times such that each flip represents one of the eight days. A result of heads indicates a day when precipitation occurs. Count the number of heads.

14. Answers will vary. One such simulation would involve writing "precipitation" on one of six identical pieces of paper and "no precipitation" on the other five pieces of paper. Select a paper at random from a container with all six pieces of paper eight times, with replacement, and record the number of times "precipitation" appears in those eight trials. Another such simulation may involve a number cube. Let one number indicate precipitation and the other five indicate no precipitation. Roll the number cube eight times for the eight days and record the number of times the precipitation number occurs.

Teacher to Teacher

For Items 14–16, note that technology or random digit tables could also be used.

For additional technology resources, visit SpringBoard Digital.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to design a simulation. Students may need extra practice matching the design to the frequency of precipitation in a given situation.

Lesson 38-2

PLAN

Materials:

- cards from Lesson 38-1
- number cubes
- random number table

Pacing: 1 class period

Chunking the Lesson

#1–2 #3–5 #6–7

Check Your Understanding

Lesson Practice

TEACH

Bell-Ringer Activity

Have students solve the following problem: If Juan makes 2 out of 3 penalty shots in his soccer match, how could you simulate this situation?

1–2 Summarizing, Paraphrasing, Create Representations, Look for a Pattern Students will create many simulations of Martin's wins and losses. They will share their sequences with their group, so that group members will end up with the same sequences to work with.

My Notes

Learning Target:

- Determine if a simulation indicates whether observed data are consistent or inconsistent with a conjecture about the data.

SUGGESTED LEARNING STRATEGIES: Close Reading, Predict and Confirm, Summarizing, Paraphrasing, Think Aloud, Debriefing, Discussion Groups

1. In the previous lesson, you carried out a simulation by mixing ten cards representing Martin's wins and losses. Next you created a sequence of the results and then computed the improvement score for the sequence you created. Repeat that process, recording below the improvement score for each randomly ordered sequence of wins and losses that you get. Work with your group and collect your results together until you have collected 40 improvement scores. (Keep all 40 sequences for use later in this activity.)

Sample collection:

0	0	0	−2	2	0	0	0
2	0	0	0	−2	0	0	0
0	0	0	0	2	−2	2	0
4	−2	0	−2	−2	2	2	−2
0	2	2	−2	0	−2	−2	−4

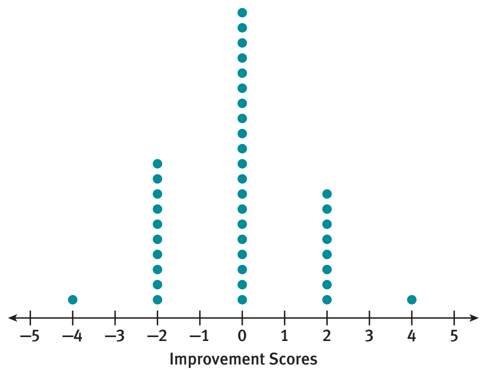
The quantities in the grid above are one example of an outcome in such a simulation.

Lesson 38-2
Confirming Data with Simulations

ACTIVITY 38
continued

My Notes

2. Make a dot plot showing the distribution of the improvement scores that you found in Item 1.



The dot plot will reflect the results of the simulation in Item 1. Most groups' dot plots will resemble a normal distribution of improvement scores as long as the simulation is done randomly.

3. Recall Martin's improvement score that you found in Item 7 of Lesson 1. Describe the column of dots in your dot plot that corresponds to Martin's improvement score.
Descriptions are likely to include the number of dots in the dot plot, a mention that the height of the column is neither the highest nor the shortest, or other reasonable observations.

4. Why are there no improvement scores of ±1, ±3, ±5?
Martin had four wins and six losses. In the two groups of five games, there will be either an even number of wins in both groups (0 and 4 or 2 and 2) or an odd number of wins in both groups (1 and 3). In either case, the difference of two even or two odd integers will always be even.

5. Based upon your results, what is the probability of Martin obtaining the improvement score that he received in his initial game? What is the probability of receiving *at least* that score?
Answers will vary, but in the sample results provided, the probability of receiving an improvement score of 2 is $\frac{8}{40} = 0.20$, a one-fifth probability. The probability of receiving at least a score of 2 (a score of 2 or 4) is $\frac{9}{40} = 0.225$.

1–2 (continued) The data collected from Item 1 are then organized in a dot plot. It would be beneficial for each group to display their results. Discuss the fact that this dot plot appears to represent a normal distribution.

Teacher to Teacher

Students should recognize that the dot plot represents a normal distribution. This is caused by random generation of values.

3–5 Quickwrite, Look for a Pattern, Create Representations, Debrief Students should be able to recognize possible values for the statistic in their simulation. They will take the initial improvement score and determine the probability of getting that score or better. For students who struggle with these items, have them go back to the chart in Item 1 and actually count the improvement scores.

6–7 Debriefing, Look for a Pattern, Self Revision/Peer Revision Students should give a detailed explanation of how they arrive at their answers. If students are divided in their decision, have a discussion to determine whether this is a valid method of determining improvement. This allows for the opportunity to explain inference and how decisions are made based on statistics that are gathered.

Teacher to Teacher

This may be a good place to discuss how other methods of simulation are carried out. For example, consider number cubes, cards, coins, or a random number table.

Connect to AP

AP Statistics students are expected to understand the logic behind "hypothesis tests." This activity involves precisely the same logic, but at a level that is more accessible using simulations to make specific determinations based on inferential statistics.

Check Your Understanding

Debrief students' answers to these items to ensure that they understand how simulations can contradict what they believe to be true. This is the basis for inference and the importance of designing an appropriate simulation. Some students may not be familiar with the concept of *proof by contradiction*. If that is the case, then show a simple example. One that can be easily found online is a proof that there is no "largest" prime number.

Answers

8. Bob could choose a random selection process (use 25 identical pieces of paper, 15 with "F" and 10 with "M" written on them and choose 11 at random; assign numbers 01 to 15 as female, 16 to 25 as male, and choose clusters of two digits from a random digits table until 11 distinct numbers are chosen; assign females 1–15 and males 16–25, then use technology to select 11 numbers between 1 and 25) and perform the simulation a relatively large number of times. He would then create a dot plot or histogram of the distribution of the number of females on the team to determine the likelihood of 11 being female.

9. The similarity in both is that there is an assumption made that might not be true. In the activity, the assumption Hannah made was that Martin's wins are in a random order and did not represent improvement. This is comparable to assuming the contradiction in the proof. The difference in these two processes is that in a proof by contradiction, you arrive at impossibility. In this activity, you arrive at an *unlikely* occurrence, not an *impossible* one.

My Notes

6. Is Martin's improvement score one that is likely to occur by chance?
Answers will vary depending on the interpretation of 0.225, the probability that represents the likelihood that a score is representative of "improvement" in the game. Students who claim this represents an "unlikely event" will be correct in determining that Martin's score does imply improvement and that Hannah's claim is rejected. However, some students may claim that 0.225 represents a probability of an event that is likely to occur (nearly 1 of every 4 trials). These students would be correct to conclude that Hannah's claim cannot be rejected. Such specific determinations cannot be made without more advanced inferential statistical methods.

7. Consider the event that Martin's sequence of game results was LLLLLLWWWW. Determine his improvement score for this game, and interpret the score with respect to Hannah's claim that his results did not indicate improvement.
Based on the results of the example experiment, only 1 of 40 trials yielded a score of 4, a probability of 0.025. Such a probability represents an event that is very unlikely to occur, so there is compelling evidence to reject Hannah's claim (much more so than with an improvement score of 2).

Check Your Understanding

8. A physical education class with 15 female students and 10 male students had to select 11 students at random to form a soccer team. Bob was skeptical when the teacher announced that all 11 players selected were female. Describe a simulation that Bob could perform that would determine if such a selection was likely a result of chance or a result of some bias.
9. One method of proof in mathematics is known as "proof by contradiction." In such proofs, you begin with a negation of the statement you wish to prove. Then, through logical deduction using known facts, a false statement is concluded. Since the conclusion is false, the original statement must be false, and the statement you want to prove is correct.

   Identify one similarity and one difference between a mathematical proof by contradiction and the logical argument that you made in Items 6 and 7.

Lesson 38-2
Confirming Data with Simulations

LESSON 38-2 PRACTICE

Consider the following alternative statistic to measure improvement: add together the position numbers of all the wins. The larger the total is, the later in the sequence the wins must be. For example:

$$\text{L, L, L, L, W, L, L, W, W, W} \rightarrow 5 + 8 + 9 + 10 = \mathbf{32}$$

$$\text{W, L, L, L, L, W, W, W, L, L} \rightarrow 1 + 6 + 7 + 8 = \mathbf{22}$$

Call this new statistic the "improvement measure." In the items that follow, use the improvement measure to see whether Martin's particular sequence of wins and losses could easily be explained by his sister Hannah's theory that his wins and losses were really just in a random order.

10. Determine Martin's improvement measure.

11. Describe how you will simulate whether or not Martin's sequence of game outcomes is consistent with Hannah's theory.

12. Show the distribution of the improvement measures that result from many random orderings of Martin's game outcomes. Use the sequences you obtained from the 40 trials in Item 1 of this lesson.

13. State a conclusion about whether Martin's sequence of wins and losses is consistent with Hannah's theory.

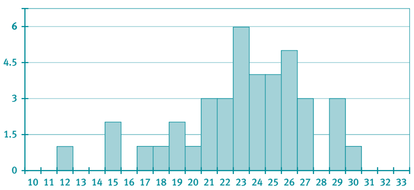
14. Explain the logic that led you to your conclusion.

My Notes

CONNECT TO AP

In AP Statistics, it is critical that students be able to write coherent and clear descriptions of simulations that even a non-statistician would be able to follow.

12. Make a dot plot or histogram of all the improvement measures. Sample histogram:



13. Hannah's theory that Martin's results do not reflect improvement is rejected.

14. Martin's improvement measure is 30 and random reordering of Martin's game outcomes resulted in an improvement measure as large as 30 only one time out of 40. This probability of 0.025 is small, indicating that, by chance, this occurrence is rare.
Answers will vary in the actual simulated numbers that students obtain, but they should find that Martin's sequence of wins and losses is *not* very consistent with Hannah's theory.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 38-2 PRACTICE

10. In Martin's LLLWLLWLWW list, the position numbers for wins are 4, 7, 9, and 10. The sum of these numbers is 30.

11. (Suggest that students use the same 40 trials that they used before in Item 1 of Lesson 2.) Take four cards representing wins and six cards representing losses, and put them in a random order. Compute the improvement statistic for that sequence of wins and losses. Repeat the process 40 times and make a dot plot of all the improvement statistics.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to design and carry out simulations. This includes being able to use the statistics gathered to make inferences.

For additional practice, have students perform simulations like cola taste tests using a random digits table or rolling number cubes.

ACTIVITY PRACTICE

1. Answers will vary. One possible solution is to take six index cards and write "girl" on three of them and "boy" on the other three. Those represent the genders of Jesse's six friends. Turn all cards face down and mix them, then put them into a group of four—those voting for Sarah—and a group of two—those voting for John.
2. Look at the cards and count how many people voted "against gender", the statistic.
3. Repeat the process, keeping track of the statistic identified in Item 2. After a relatively large number of simulations (maybe 50 or more), make a graphic display of the results. Then identify the probability of the occurrence of Jesse's friends' statistic based on the distribution.
4. Since a probability of 0.40 indicates a likely result, then the occurrence can be the result of chance. The conclusion would be that Jesse's theory that his friends were voting according to gender is rejected.
5. Since a probability of 0.05 indicates an unlikely result, then the occurrence is probably not the result of chance. The conclusion would be that Jesse's theory that his friends were voting according to gender is not rejected.
6. The statistic being measured is the number of subjects who identified the different cola correctly.
7. One-third would likely choose correctly just by guessing, so there is an expectation of six or seven of the 20 subjects giving correct identifications if there was no difference in taste.
8. Sample answer: Each roll of the number cube represents one subject. Two numbers on the cube represent a successful identification and four numbers represent an unsuccessful identification. Roll the cube twenty times and count the number of successes. Then repeat this many times and make a graphic display of the distribution of the number of successes. Finally, determine if 12 successes looked likely based on the distribution. If the occurrence of 12 successes seems unlikely, conclude that some people are not just guessing, but can really taste the difference between the drinks.

ACTIVITY 38 PRACTICE

**Write your answers on notebook paper.
Show your work.**

Use the following information for Items 1–5.

Jesse, a high school junior, was talking with six of his friends about whom they planned to vote for in the upcoming election of the class president. There were two candidates, Sarah and John. Among Jesse's group of friends there were three girls, and all of them planned to vote for Sarah, a girl. Jesse's three other friends were boys, and two of them planned to vote for John, a boy. Only one friend of Jesse's—a boy—was planning to vote "against gender" and vote for Sarah. Jesse thought that his friends were voting according to their own gender and wondered if this was just a chance occurrence.

1. Jesse wants to perform a simulation to determine if his friends' tendency to vote according to gender was likely a result of random chance. Describe (but do not perform) a simulation that Jesse could perform to accomplish this task.
2. Identify a statistic that Jesse could measure in his simulation.
3. Describe the process for determining the likelihood of the occurrence of the statistic for Jesse's friends.
4. Based on your results from Item 3, assume that the probability of the occurrence of the statistic was 0.40. What conclusion would you make?
5. Based on your results from Item 3, assume that the probability of the occurrence of the statistic was 0.05. What conclusion would you make?

Use the following information for Items 6–9.

For a research project, Tia wanted to see whether people could tell the difference between two brands of cola by taste. She planned an experiment. Volunteer subjects would each be presented with three small identical-looking cups of soda labeled A, B, and C. Two of the cups would contain the same brand of cola while the third cup would contain the other brand. Tia would randomly determine which of the three cups would be the one containing the different brand. She would also randomly determine which cola brand would be in two cups and which would be in one cup.

Each subject would be asked to taste the cola in each cup and then identify which cup contained the different brand. The subjects would not be required to identify the brands, only to tell which cup contained a different brand.

After getting responses from 20 subjects, Tia planned to count how many had identified the correct cup, and then see whether that count was too large to be explainable by just random chance.

6. Identify the statistic that Tia is measuring.
7. Tia is interested in seeing whether her statistic is greater than she would expect by chance alone. What would the value of her statistic be if no one could taste a difference between the two drinks?

Use the following information for Items 8 and 9.

Suppose that 12 of the 20 people in Tia's experiment gave correct cup identifications. Describe a process by which Tia could decide whether 12 correct cup identifications would or would not be surprising if, in fact, everyone was just guessing.

8. Describe such a process using a six-sided number cube. Be sure to identify what each roll of the number cube represents and what the numbers on the number cube represent. You do not have to carry out the process—just describe it clearly.

MATHEMATICAL PRACTICES
Make Sense of Problems and Persevere in Solving Them

9. Describe another such process using only a random number table. Be sure to identify what each digit represents and the meaning of that digit.

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9. Let each non-zero digit represent a subject in the study. Since random guessing would lead people to get it right one-third of the time, let one-third of the digits represent successful identifications. Let the digits 1–3 represent successes, and let the digits 4–9 represent incorrect cup identifications. Ignore all zeros. In this manner, every subject has a one-third probability of success by chance.

   Begin at a random location on the random digits table. Using the first 20 non-zero digits, count the number of 1s, 2s, and 3s that appear. Continue in the random digits table, repeating the process with the next 20 non-zero digits.

   Repeat several times and make a graphic display of the distribution of the number of successes. Finally, determine if 12 successes is likely based on the distribution. If the occurrence of 12 successes seems unlikely, conclude that some people are not just guessing, but can really taste the difference between the drinks.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.